

# Exact inhomogeneous models and the drift of light rays induced by nonsymmetric flow of the cosmic medium

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After introducing the Szekeres and Lemaître–Tolman cosmological models, the real-time cosmology program is briefly mentioned. Then, a few widespread misconceptions about the cosmological models are pointed out and corrected. Investigation of null geodesic equations in the Szekeres models shows that observers in favourable positions would see galaxies drift across the sky at a rate of up to  $10^{-6}$  arc seconds per year. Such a drift would be possible to measure using devices that are under construction; the required time of monitoring would be  $\approx 10$  years. This effect is zero in the FLRW models, so it provides a measure of inhomogeneity of the Universe. In the Szekeres models, the condition for zero drift is zero shear. But in the shearfree normal models, the condition for zero drift is that, in the comoving coordinates, the time dependence of the metric completely factors out.

## I. INHOMOGENEOUS MODELS IN ASTROPHYSICS<sup>1</sup>

Just as was the case with our earlier reviews [1–3], we define inhomogeneous cosmological models as those exact solutions of Einstein’s equations that contain at least a subclass of nonvacuum and nonstatic Friedmann – Lemaître – Robertson – Walker (FLRW) solutions as a limit. The reason for this choice is that such FLRW models are generally considered to be a good first approximation to a description of our real Universe, so it makes sense to consider only those other models that have a chance to be a still better approximation. Models that do not include an FLRW limit would not easily fulfil this condition.

This is a live topic, with new contributions appearing frequently, so any review intended to be complete would become obsolete rather soon. Ref. 1 is complete until 1994, Ref. 2 is a selective update until 2009 and Ref. 3 is a more selective update until the end of 2010. The criterion of the selection in the updates was the usefulness of the chosen papers for solving problems of observational cosmology.

In the very brief overview given here we put emphasis on pointing out and correcting the erroneous results that exist in the literature and are being taken as proven truths. Some of them have evolved to become research paradigms, with many followers, some others proceed in this direction. We hope to stop this process, which disturbs and slows down the recognition of the inhomoge-

neous models as useful devices for understanding the observed Universe.

We first present the two classes of inhomogeneous models that, so far, proved most fruitful in their astrophysical application: the Szekeres model [4], and its spherically symmetric limit, the Lemaître [5] – Tolman [6] (L–T) model. Then we briefly mention the real-time cosmology program, and we give an overview of the erroneous ideas. Finally, we present an effect newly calculated in a few classes of models that is possible to observe and can become a test of homogeneity of the Universe: the drift of light rays induced by nonsymmetric flow of the cosmic medium

## II. THE SZEKERES SOLUTION

The (quasi-spherical) Szekeres solution [4, 7] is, in comoving coordinates

$$ds^2 = dt^2 - \frac{\mathcal{E}^2(\Phi/\mathcal{E})_{,r}^2}{1 + 2E(r)} dr^2 - \frac{\Phi^2}{\mathcal{E}^2} (dx^2 + dy^2),$$

$$\mathcal{E} \stackrel{\text{def}}{=} \frac{(x - P)^2}{2S} + \frac{(y - Q)^2}{2S} + \frac{S}{2}, \quad (2.1)$$

where  $E(r)$ ,  $M(r)$ ,  $P(r)$ ,  $Q(r)$  and  $S(r)$  are arbitrary functions and  $\Phi(t, r)$  obeys

$$\Phi_{,t}^2 = 2E(r) + \frac{2M(r)}{\Phi} + \frac{1}{3}\Lambda\Phi^2. \quad (2.2)$$

The source in the Einstein equations is dust, whose mass density in energy units is

$$\kappa\rho = \frac{2(M/\mathcal{E}^3)_{,r}}{(\Phi/\mathcal{E})^2(\Phi/\mathcal{E})_{,r}}. \quad (2.3)$$

<sup>1</sup> This article is based on the talk delivered at the Marcel Grossman Meeting 13 in Stockholm, 2012.

Eq. (2.2) implies that the bang time is in general position-dependent:

$$\int_0^\Phi \frac{d\tilde{\Phi}}{\sqrt{2E + 2M/\tilde{\Phi} + \frac{1}{3}\Lambda\tilde{\Phi}^2}} = t - t_B(r). \quad (2.4)$$

The general Szekeres metric has no symmetry. It contains the spherically symmetric Lemaitre [5] – Tolman [6] (L–T) model as the limit of  $(P, Q, S)$  being all constant. The latter is usually used with a different parametrisation of the spheres of constant  $(t, r)$ , namely

$$ds^2 = dt^2 - \frac{R_{,r}^2}{1 + 2E} dr^2 - R^2(t, r) (d\vartheta^2 + \sin^2 \vartheta d\varphi^2), \quad (2.5)$$

where  $R \equiv \Phi$ , still obeying (2.2), and (2.3) simplifies to

$$\kappa\rho = \frac{2M_{,r}}{R^2 R_{,r}}. \quad (2.6)$$

The Friedmann limit follows when, in addition,  $\Phi(t, r) = rS(t)$ ,  $2E = -kr^2$  where  $k = \text{const}$  is the FLRW curvature index, and  $t_B$  is constant.

### III. REAL-TIME COSMOLOGY

There are ways in which the expansion of the Universe might be directly observed. The authors of Refs. [8, 9] composed them into a paradigm termed *real-time cosmology*. One of them is the *redshift drift*: the change of redshift with time for a fixed light source, induced by the expansion of the Universe.

Consider, as an example, an L–T model with  $\Lambda$ , given by (2.5) with  $R$  obeying (2.2). Along a single radial null geodesic, directed toward the observer,  $t = T(t_o, r)$  (where  $t_o$  is the instant of observation) the redshift is [10]

$$1 + z(t_o, r) = \exp \left[ \int_{r_{\text{em}}}^{r_{\text{obs}}} \frac{R_{,tr}(T(t_o, r), r)}{\sqrt{1 + 2E(r)}} dr \right]. \quad (3.1)$$

For any fixed source (i.e. constant  $r$ ), eq. (2.2) defines a different expansion velocity  $R_{,t}$  for  $\Lambda = 0$  and for  $\Lambda \neq 0$ , and thus allows us to calculate the contribution of  $\Lambda$  to  $z$  via (3.1). With the evolution type known, the expansion velocity depends on  $r$ , and so allows us to infer the distribution of mass along the past light cone.

According to the authors of [8, 9], the “European Extremely Large Telescope” (E-ELT)<sup>2</sup> could detect the redshift drift during less than 10 years of monitoring a given light source. The Gaia observatory<sup>3</sup> could achieve this during about 30 years.

For more on redshift drift see the contribution by P. Mishra, M.-N. C  l  rier and T. Singh in these Proceedings.

The drift of light rays described in Sec. V and following is another real-time cosmology effect.

### IV. ERRONEOUS IDEAS AND PARADIGMS

The L–T model was noticed in the astrophysics community – but:

1. Many astrophysicists treat it as an enemy to kill rather than as a useful new device. (Citation from Ref. [9]: The Gaia or E-ELT projects could distinguish FLRW from L–T “possibly eliminating an *exotic alternative explanation to dark energy*”).

2. Some astrophysicists practise a loose approach to mathematics. An extreme example is to take for granted every equation found in any paper, without attention being paid to the assumptions under which it was derived.

Papers written in such a style planted errors in the literature, which then came to be taken as established facts. In this section a few characteristic errors are presented (marked by ●) together with their explanations (marked by \*).

● The accelerating expansion of the Universe is an observationally established fact (many refs., the Nobel Committee among them).

\* The established fact is the *smaller than expected observed luminosity of the SNIa supernovae* (but even this is obtained assuming that FLRW is the right cosmological model). **The accelerating expansion is an element of theoretical explanation of this observation.** When the SNIa observations are interpreted against the background of a suitably adjusted L–T model, they can be explained by matter inhomogeneities along the line of sight, with decelerating expansion [11, 12].

● Positive sign of the redshift drift is a direct confirmation of accelerated expansion of the space.

\* The sign of redshift drift is only related to acceleration when homogeneity is assumed [13] (see also Mishra, C  l  rier, and Singh in these Proceedings).

●  $H_0$ , supernovae and the cosmic microwave background radiation (i.e. the size of the sound horizon and the location of the acoustic peaks) are sufficient to rule out inhomogeneous L–T models.

\* These observations only depend on  $D(z)$  and  $\rho(z)$ , and thus can be accommodated by the L–T model, which is specified by 2 arbitrary functions. Examples of such constructions are given in [12, 14]. In fact, as follows from the Sachs equations, in the approximation of small null shear (which for most L–T models works quite well [15]),

<sup>2</sup> Now in the planning, to be built in Chile.

<sup>3</sup> <http://sci.esa.int/science-e/www/area/index.cfm?fareaid=26>

there is a relation between  $D(z)$ ,  $\rho(z)$  and  $H(z)$ , meaning that these 3 observables are not independent[15], and thus allow the L-T model to accommodate more data on the past null cone.

● The gravitational potential of a typical structure in the Universe is of the order of  $10^{-5}$  and thus, by writing the metric in the conformal Newtonian gauge, one immediately shows that inhomogeneities can only introduce minute ( $\approx 10^{-5}$ ) deviations from the RW geometry. This also means that the evolution of the Universe must be Friedmannian (common argument among cosmologists).

\* Even if the gravitational potential remains small, its spatial derivatives do not, and thus the model has completely different optical properties than the Friedmann models. This was shown by rewriting one of the L-T Gpc-scale inhomogeneous models in the conformal Newtonian coordinates [16]. The gravitational potential of this model remains small yet the distance-redshift relation deviates strongly from that for the background model.

The question remains whether small-scale fluctuations (of the order of tens of Mpc) could also modify optical properties and evolution of the Universe. The problem is complicated as it requires solving the Einstein equations and null geodesics for a general matter distribution, which, with current technology, is not possible to do numerically. Therefore, the problem has been addressed in a number of approximations and toy models. Recent studies showed that the optical properties along a single line of sight can be significantly different than in the Friedmann model. Yet, if averaged over all directions, the average distance-redshift relation closely follows that of the model, which describes the evolution of the average density and expansion rate[15]. Thus, the problem reduces to the following one: do the average density and expansion rate follow the evolution of the homogeneous model, i.e. is the evolution of the background affected by small-scale inhomogeneities and does it deviate from the Friedmannian evolution? Some authors claim that matter inhomogeneities cannot affect the background and that the Universe must have Friedmannian properties[17, 18], while others argue for strong deviation from the Friedmannian evolution[19, 20]. Studies of this problem within the exact models, like L-T, proved that under certain conditions the back-reaction can be large, while under others it remains quite small [21], leaving the problem unsolved. For an informative description of the problem and techniques used to address it see Ref. [22, 23].

● Fitting an L-T model to number counts or the  $D_L(z)$  relation results in predicting a huge void, several hundred Mpc in radius, around the centre (too many papers to be cited, literature still growing). Measurements of the dipole component of the CMB radiation then imply that our Galaxy should be very close to the center of this void, which contradicts the “cosmological principle”.

\* *The implied huge void is a consequence of handpicked constraints imposed on the L-T model*, for example constant  $t_B$ . When the model is employed at full generality, the giant void is not implied [14].

\* *The cosmological principle is a postulate, not a law of Nature*, it cannot say which model is “right” and which is “wrong”.

● The bang time function must be constant, otherwise the decaying mode of density perturbation is nonzero, which implies large inhomogeneities in the early universe (see, for example, Ref. [24]).

\* The bang time function describes the differences in the age between different regions of the Universe. In the L-T and Szekeres models it is also related to the amplitude of the decaying mode. However, these models describe the evolution of dust and therefore cannot be extended to times before the recombination, when the Universe was in a turbulent state: rotation, plasma, pressure gradients all did affect the proper time of an observer ( $d\tau = dt \sqrt{g_{00}(t, x^i)}$ ). Eventually, even if the Universe started with a simultaneous big bang, by the time of recombination, due to the standard physical processes, the age of the Universe would have been different at different spatial positions, giving rise to non-constant  $t_B(r)$  of the dust L-T or Szekeres model that takes over there.

Moreover, the relation between the nonsimultaneous big bang and the decaying mode was established only for the L-T [7, 25] and Szekeres [26] models. For more general models, not yet explicitly known as solutions of Einstein’s equations, like the ones mentioned above, the connection may be more complicated and indirect. Thus, citing this relation for such a general situation is an illegitimate stretching of a theorem beyond the domain of its assumptions (see also the next entry below).

● The L-T models used to explain away dark energy must have their bang-time function constant, or else they “can be ruled out on the basis of the expected cosmic microwave background spectral distortion” [27].

\* *The papers that claim this parametrise their models with a set of very simple functions, which lack flexibility*. This sets them on the wrong track from the beginning.

\* *In order to meaningfully test any cosmological model against observations, one must apply it at every step of analysis* of the observational data. To do so, would require a re-analysis of a huge pool of data. C. Hellaby with coworkers [28] is working on such a program applied to the L-T model, but the work is far from being completed.

Lacking any better chance, we currently use observations interpreted in the FLRW framework to infer about the  $M(r)$  and  $t_B(r)$  functions in the L-T model. This is justified as long as we intend to point out possibilities,

under the tacit assumption that these results will be verified in the future within a complete revision of the observational material on the basis of the L–T model. However, putting “precise” bounds on the L–T model functions using the self-inconsistent mixture of FLRW/L–T data available today is a self-delusion. An example: the spatial distribution of galaxies and voids is inferred from the luminosity distance vs. redshift relation that applies *only* in the FLRW models. Without assuming the FLRW background, we know nothing about this distribution until we reconstruct it using the L–T model from the beginning.

The L–T and Szekeres models cannot be treated as exact models of the Universe, to be taken literally in all their aspects. They are *exact as solutions of Einstein’s equations*, but when applied in cosmology, they are merely *the next step of approximation after FLRW*. If the FLRW approximation is good for some purposes, then a more detailed model, *when applied in a situation, in which its assumptions are fulfilled*, can only be better.

## V. THE REDSHIFT EQUATIONS IN THE SZEKERES MODELS

Consider two light rays, the second one following the first after a short time-interval  $\tau$ , both emitted by the same source and arriving at the same observer. The trajectory of the first ray is given by

$$(t, x, y) = (T(r), X(r), Y(r)), \quad (5.1)$$

the corresponding equation for the second ray is

$$(t, x, y) = (T(r) + \tau(r), X(r) + \zeta(r), Y(r) + \psi(r)). \quad (5.2)$$

This means that while the first ray intersects a hypersurface  $r = r_0$  at  $(t, x, y) = (T, X, Y)$ , the second ray intersects the same hypersurface not only later, but, in general, at a different comoving location.  $\Rightarrow$  ***In general the two rays will intersect different sequences of intermediate matter worldlines.***

The same is true for nonradial rays in the L–T model.

Consequently, the second ray is emitted in a different direction and is received from a different direction by the observer. Thus, a typical observer in a Szekeres spacetime should see each light source slowly ***drift across the sky***. How slowly will be estimated further on. As will be seen from the following, ***the absence of this drift is a property of exceptionally simple geometries*** (or exceptional directions in more general geometries).

We assume that  $(\zeta, \psi)$  and  $(d/dr)(\tau, \zeta, \psi)$  are small of the same order as  $\tau$ , so we neglect all terms nonlinear in any of them and terms involving their products.

For any function  $f(t, r, x, y)$  the symbol  $\Delta f$  will denote

$$f(t + \tau, r, x + \zeta, y + \psi) - f(t, r, x, y) \quad (5.3)$$

***linearized in  $(\tau, \zeta, \psi)$*** . Note: ***the difference is taken at the same value of  $r$*** . Applying  $\Delta$  to the null

geodesic equations parametrised by  $r$  we obtain the equations of propagation of  $(\tau, \zeta, \psi)$  and  $(\xi, \eta) \stackrel{\text{def}}{=} (d/dr)(\zeta, \psi)$  along a null geodesic – see both sets of equations fully displayed in Ref. [29].

## VI. REPEATABLE LIGHT PATHS

There will be no drift when, for a given source–observer pair, each light ray will proceed through the same intermediate sequence of matter world lines. Rays having this property will be called ***repeatable light paths (RLP)***. For a RLP we have

$$\zeta = \psi = \xi = \eta = 0 \quad (6.1)$$

all along the ray. The equations of propagation of  $(\tau, \zeta, \psi, \xi, \eta)$  become then overdetermined (3 equations to determine the propagation of  $\tau$  along a null geodesic), and imply limitations on the metric components. They can be used in 2 ways:

1. As the condition (on the metric) for ***all*** null geodesics to be RLPs.
2. As the conditions under which special null geodesics are RLPs in subcases of the Szekeres spacetime.

In the first interpretation, the equations of propagation should be identities in the components of  $dx^\alpha/dr$ , and this happens when

$$\Psi \stackrel{\text{def}}{=} \Phi_{,tr} - \Phi_{,t} \Phi_{,r} / \Phi = 0. \quad (6.2)$$

This means zero shear, i.e. the Friedmann limit. Thus, we have the following

**Corollary:**

***The only spacetimes in the Szekeres family in which all null geodesics have repeatable paths are the Friedmann models.***

In fact, we proved something stronger. Since the drift vanishes in the Friedmann models, we have:

**Corollary 2:**

***The presence of the drift would be an observational evidence for the Universe to be inhomogeneous on large scales.***

In the second interpretation, there are only 2 nontrivial (i.e. non-Friedmannian) cases:

A. When the Szekeres spacetime is axially symmetric ( $P$  and  $Q$  are constant). In this case, the RLPs are those null geodesics that stay on the axis of symmetry in each 3-space of constant  $t$ .

B. When the Szekeres spacetime is spherically symmetric ( $P, Q, S$  are all constant) – then it reduces to the L–T model. In this case, the radial null geodesics are ***the only*** RLPs that exist. A formal proof of this statement is highly complicated [29].

For non-radial rays in the L–T model, the non-RLP phenomenon was predicted, by a different method (and under the name of ***cosmic parallax***), by Quercellini *et al.* [8, 9]. Their review contains a broad presentation of the real-time cosmology paradigm, oriented toward observational possibilities.

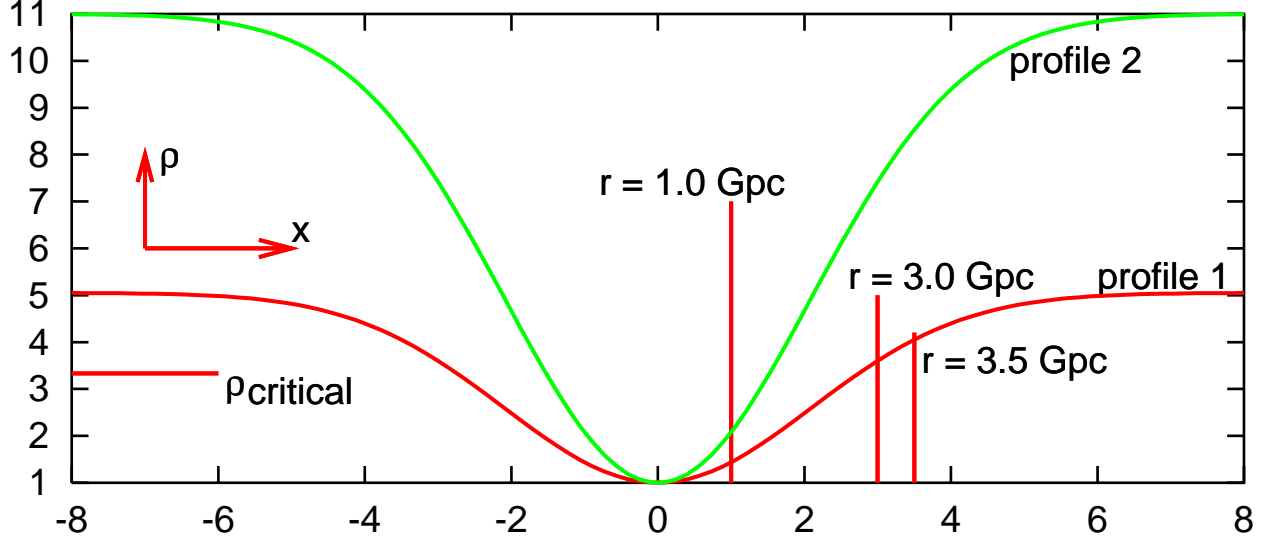


FIG. 1: Density profiles and positions of the observers used in the numerical examples.

## VII. EXAMPLES OF NON-RLPS IN THE L-T MODEL

The examples will show the non-RLP effect for non-radial null geodesics in two configurations of the L-T model, shown in Fig. 1, for different positions of the observers with respect to the center of symmetry. In Example 1 (see Fig. 2) we use Profile 1, the observer and the light source at 3.5 Gpc from the center of the void, the directions to them at the angle 1.8 rad, and [30]

$t_B = 0$  – simultaneous Big Bang,

$\rho(t_0, r) = \rho_0 [1 + \delta - \delta \exp(-r^2/\sigma^2)]$  – the density profile at the current instant,

$r \stackrel{\text{def}}{=} R(t_0, r)$  – the radial coordinate,

$\rho_0 \stackrel{\text{def}}{=} \rho(t_0, 0) = 0.3 \times (3H_0^2)/(8\pi G) \equiv 0.3\rho_{\text{critical}}$  – the present density at the center of the void,

$H_0 = 72 \text{ km s}^{-1} \text{ Mpc}^{-1}$  – the present value of the Hubble parameter,

$\delta = 4.05, \sigma = 2.96 \text{ Gpc}$ .

The source in Fig. 2 sends three light rays to the same observer, the first of which was received by the observer  $5 \times 10^9$  years ago, the second is being received right now, and the third one will be received  $5 \times 10^9$  years in the future. Fig. 3 shows these rays projected on the space  $t = \text{now}$  along the flow lines of the L-T dust. The source is at the upper right corner of the graph and the observer is at the upper left corner.

The *time-averaged* rate of change of the position of the source in the sky, seen by the observer is

$$\dot{\gamma} = \frac{\text{angle between the earlier and the later ray}}{\text{time interval} = 5 \times 10^9 \text{ years}} \sim 10^{-7} \frac{\text{arcsec}}{\text{year}}. \quad (7.1)$$

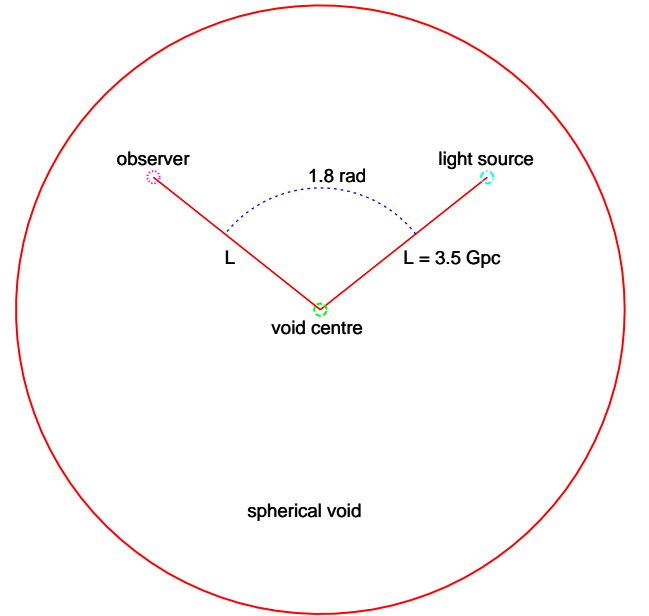


FIG. 2: The configuration of the light source and the observer in example 1.

The rate of drift in the next figures is calculated in the same way.

In Examples 2, 3 and 4 (Fig. 4), the observer O is at  $R_0$  from the center; the angle between the direction toward the galaxy ( $*$ ) and toward the origin is  $\gamma$ . For each  $\gamma$  we calculated the rate of change  $\dot{\gamma}$  by eq. (7.1), and the graphs in Fig. 5 show  $\dot{\gamma}$  as a function of  $\gamma$ . All the examples have  $d = 1 \text{ Gly} \approx 306.6 \text{ Mpc}$ . Example 2 (solid line) has  $R_0 = 3 \text{ Gpc}$  and Profile 1; Example 3

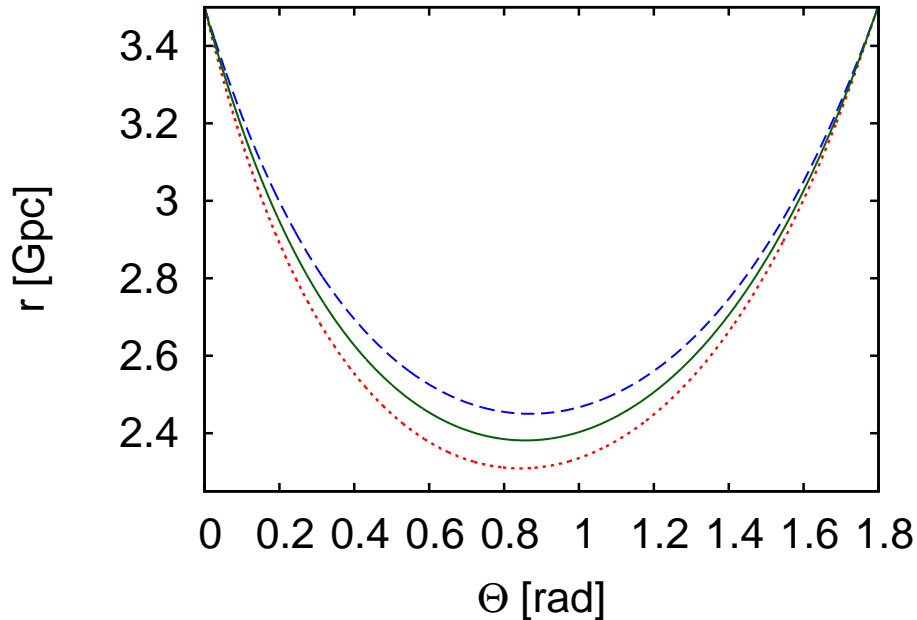


FIG. 3: Three light rays projected on the space of constant comoving time along the flow lines of the cosmic medium. Middle line: the ray received at the current instant; upper line: the ray received  $5 \times 10^9$  years ago; lower line: the ray to be received  $5 \times 10^9$  years in the future.

(dashed line) has  $R_0 = 1$  Gpc and Profile 1; Example 4 (dotted line) has  $R_0 = 1$  Gpc and Profile 2 (for which  $\delta = 10.0$  instead of  $\delta = 4.05$ ; this is a deeper void in a higher-density background). The amplitude is  $\sim 10^{-7}$  for (3) and  $\sim 10^{-6}$  for (2) and (4). With the Gaia accuracy of  $5 - 20 \times 10^{-6}$  arcsec, we would need a few years to detect this effect.

### VIII. RLPS IN SHEARFREE NORMAL MODELS

In the Szekeres models, the condition for all null geodesics to be RLPs was the vanishing of shear. This suggests that the cause of the non-RLP phenomenon might be shear in the cosmic flow. To test this supposition, the existence of RLPs was investigated in those cosmological models in which shear is zero [31] – the shear-free normal models found by Barnes [32]. They obey the Einstein equations with a perfect fluid source and contain, as the acceleration-free limit, the whole FLRW family.

There are four classes of them: the Petrov type D metrics that are spherically, plane and hyperbolically symmetric, and the conformally flat metric found earlier by Stephani [33]. In the Petrov type D case, the metric in comoving coordinates is

$$ds^2 = \left( \frac{FV_{,t}}{V} \right)^2 dt^2 - \frac{1}{V^2} (dx^2 + dy^2 + dz^2), \quad (8.1)$$

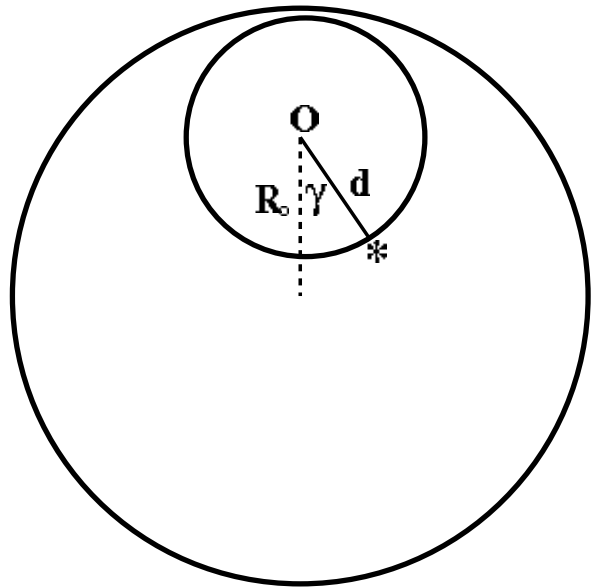


FIG. 4: The configuration for Examples 2, 3, 4.

where  $F(t)$  is an arbitrary function, related to the expansion scalar  $\theta$  by  $\theta = 3/F$ . The Einstein equations reduce to the single equation:

$$w_{,uu}/w^2 = f(u), \quad (8.2)$$

where  $f(u)$  is an arbitrary function, while  $u$  and  $w$  are

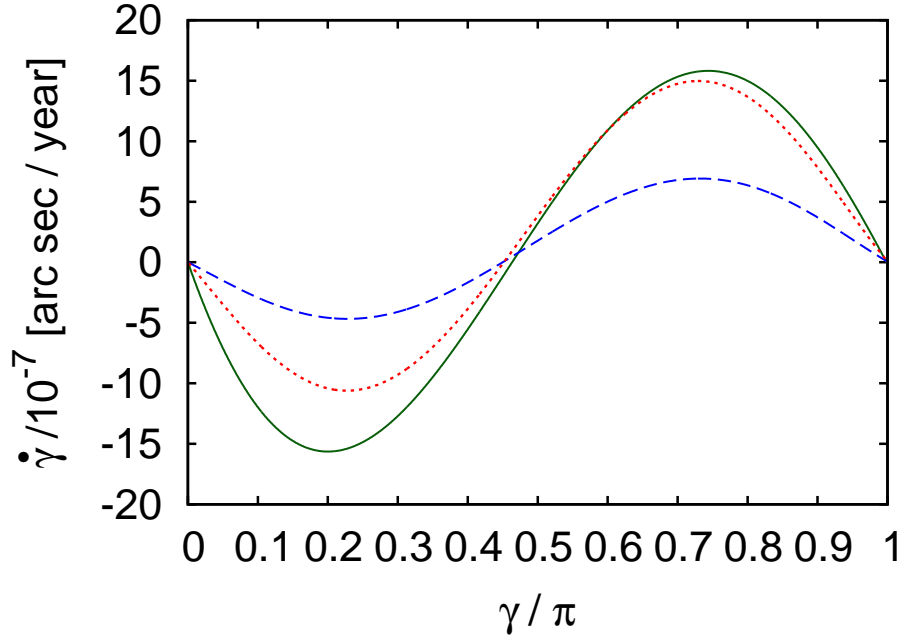


FIG. 5:  $\dot{\gamma}$  as a function of  $\gamma$  for Examples 2, 3, 4, in arcsec/(year  $\times 10^7$ ).

related to  $(x, y, z)$  and to  $V(t, x, y, z)$  differently in each subfamily. We have

$$(u, w) = \begin{cases} (r^2, V) & \text{with spherical symmetry,} \\ & r^2 \stackrel{\text{def}}{=} x^2 + y^2 + z^2; \\ (z, V) & \text{with plane symmetry;} \\ (x/y, V/y) & \text{with hyperbolic symmetry.} \end{cases} \quad (8.3)$$

The FLRW limit follows when  $f = 0$  and  $V = R(t)g(x, y, z)$ .

The conformally flat Stephani solution [1, 33] has the metric given by (8.1), the coordinates are comoving, and  $V(t, x, y, z)$  is given by

$$V = \frac{1}{R} \left\{ 1 + \frac{1}{4}k(t) \left[ (x - x_0(t))^2 + (y - y_0(t))^2 + (z - z_0(t))^2 \right] \right\}, \quad (8.4)$$

where  $(R, k, x_0, y_0, z_0)$  are arbitrary functions of  $t$ . This is a generalisation of the whole FLRW class, which results when  $(k, x_0, y_0, z_0)$  are all constant. In general, (8.4) has no symmetry.

In these models, in the most general cases, generic null geodesics are not RLPs. Consequently, *it is not shear that causes the non-RLP property*.<sup>4</sup>

In the general type D shearfree normal models, the only RLPs are radial null geodesics in the spherical case and their analogues in the other two cases. In the most general Stephani spacetime, RLPs do not exist. In the axially symmetric subcase of the Stephani solution the RLPs are those geodesics that intersect the axis of symmetry in every space of constant time. In the spherically-, plane- and hyperbolically symmetric subcases, the RLPs are the radial geodesics.

The completely drift-free subcases are conformally flat, but more general than FLRW. Their defining property is that their time-dependence in the comoving coordinates can be factored out, and the cofactor metric is static. The FLRW models have the same property. For example, in the drift-free spherically symmetric type D case:

$$ds^2 = \frac{1}{V^2} \left\{ [(A_1 + A_2 r^2) (FS, dt)]^2 - dr^2 - r^2 (d\vartheta^2 + r^2 \sin^2 \vartheta d\varphi^2) \right\}, \quad (8.5)$$

the whole non-staticity is contained in  $V$ :

$$V = B_1 + B_2 r^2 + (A_1 + A_2 r^2) S(t). \quad (8.6)$$

The  $(A_1, A_2, B_1, B_2)$  are arbitrary constants and  $S(t)$  is an arbitrary function. This model is more general than FLRW because the pressure in it is spatially inhomogeneous. The FLRW limit follows when  $A_1 \neq 0$  and  $B_2 = (A_2/A_1)B_1$ .

<sup>4</sup> Contrary to what the authors of Ref. [9] claim throughout their paper.

## IX. DEPENDENCE OF RLPS ON THE OBSERVER CONGRUENCE

The RLPs are defined relative to the congruence of worldlines of the observers and light sources. So far, we have considered observers and light sources attached to the particles of the cosmic medium, whose velocity field is defined by the spacetime geometry via the Einstein equations. But we could as well consider other timelike congruences, or spacetimes in which no preferred timelike congruence exists, for example Minkowski. It turns out that even in the Minkowski spacetime one can devise a timelike congruence that will display the non-RLP property [34].

Take the Minkowski metric in the spherical coordinates

$$ds^2 = dt'^2 - dr'^2 - r'^2 (d\vartheta^2 + \sin^2 \vartheta d\varphi^2), \quad (9.1)$$

and carry out the following transformation on it:

$$t' = (r - t)^2 + 1/(r + t)^2, \quad r' = (r - t)^2 - 1/(r + t)^2. \quad (9.2)$$

The result is the metric

$$ds^2 = \frac{1}{(r + t)^4} \left\{ 16u (dt^2 - dr^2) - (u^2 - 1)^2 (d\vartheta^2 + \sin^2 \vartheta d\varphi^2) \right\}, \quad u \stackrel{\text{def}}{=} r^2 - t^2. \quad (9.3)$$

Now we assume that the curves with the unit tangent vector field  $u^\alpha = [(r + t)^2 / (4\sqrt{u})] \delta^\alpha_0$  are world lines of test observers and test light sources.

Proceeding as before we conclude that, with respect to this congruence, generic null geodesics in the Minkowski spacetime have non-repeatable paths. (The exception are those rays that are radial in the coordinates of (9.4)). This is because the time-dependence of (9.4) cannot be factored out.

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